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ABSTRACT

High school students (N=213) of various ability levels enrolled in first-year algebra classes were presented lessons concerning geometry concepts. The lessons were varied according to the degree of complexity of the examples that were presented. Students then were tested over their understanding of the concepts. The test questions ranged from straightforward application of the lesson material to multi-step problems with high computational complexity. Significant main effects were found due to level of test question complexity, level of lesson complexity, and student ability level. Significant interactions also were identified. These results are discussed in terms of planning and presenting lessons in mathematics. (Author/PK)

The Effect of Complexity of Lesson Presentation on Student Achievement in Mathematics

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Running Head: Complexity of Lesson

Abstract

High school students ($\underline{n}$ = 213) of various ability levels were presented lessons concerning geometry concepts. The lessons were varied according to the degree of complexity of the examples that were presented. Students then were tested over their understanding of the concepts. The test questions ranged from straightforward application of the lesson material to multi-step problems with high computational complexity. Significant main effects were found due to level of test question complexity, level of lesson complexity, and student ability level. Significant interactions also were identified. These results are discussed in terms of planning and presenting lessons in mathematics.

The Effect of Complexity of Lesson Presentation on Student Achievement in Mathematics

The National Council of Teachers of Mathematics (1980) stated that problem solving should be a major focus of mathematics in the 1980's. In response, literally hundreds of studies have been conducted in an attempt to determine ways to improve students' problem solving skills. Lochhead (1981) and Silver and Thompson (1984) presented comprehensive reviews of the many directions such research has taken.

The focus of the present study concerns the level of complexity of a secondary school mathematics lesson presentation and its relation to subsequent student ability to solve problems. Typically, high school mathematics textbooks contain a variety of exercises at various levels of difficulty or complexity. It is up to the teacher to select examples to present to the class and then to select problems to assign for practice or homework. Should the teacher select the same examples and problems for students of average ability as for students of high ability? Can the teacher expect transfer of skills attained by working problems at one complexity level to those skills required at another complexity level? This study is an attempt to guide the thinking of teachers as they address questions such as those posed above.

Method

Subjects

The subjects were 222 students enrolled in first year algebra classes in two Georgia high schools. A total of 116 of the students were females and approximately 75% of the students were of Caucasian ancestry. The students represented a wide range of mathematics ability levels.

Procedure

To determine the ability levels of the students, a test composed of 20 mathematics problems from previous SAT tests was administered. Students ($\underline{n}$ = 222) were given 50 minutes to complete this test. The mean of the test was 10.10 and the standard deviation was 3.01. The split-half reliability of the test was .82. Of the 222 students, 64 made scores that were two-thirds of a standard deviation or higher above the mean. These students comprised the group referred to as "above average." Their raw scores on the test were 12 or higher. A total of 78 students scored from one-third of a standard deviation below the mean to one-third of a standard deviation above the mean. These students were referred to as "average." Their scores ranged from 9 to 11 inclusive. Eighty students made scores that were two-thirds of a standard deviation or lower below the mean. These students made up the "below average" group. Their scores were 8 or below.

The 64 students in the above average group were each randomly assigned to one of two lesson complexity conditions (high complexity, low complexity), thus dividing the above average group into two subgroups each containing

32 students. Similarly, the 78 students in the average group were each randomly assigned to one of the two complexity conditions, creating two subgroups of 39 students. Finally, the same procedure was used to assign the below average students to the two complexity groups, forming subgroups of 40 students.

One week after the 20-item test was administered, students were placed in their assigned groups and were presented a geometry lesson. On the day of the presentation, nine of the 222 students were not present. Three of those absent were in the above average group, four absentees were in the average group and two of the absentees were in the below average group. Therefore, the total number of students in the study was 213.

The geometry lesson focused on chords, tangents, and secants of circles. The content was chosen because it is covered in the geometry course regularly taught in the high schools, and because first-year algebra students would probably not have had prior exposure to such contents. Briefly, the lesson concerned three theorems. The first theorem states that "when two chords intersect inside a circle, the product of the segments of one chord equals the product of the segments of the other." The second theorem says that "when two secants are drawn to a circle from an outside point, the product of one secant and its external segment equals the product of the other secant and its external segment." The third theorem states that "when a tangent and a secant are drawn to a circle from an outside point, the square of the tangent is equal to the product of the secant and its external segment." The theorems were not proved during the lesson presentations. Instead, the lessons were begun with definitions of line segments, chords, tangents, secants, and external segments of secants. Next, each theorem was explained and three

applications of each theorem were presented. Applications involved solving for lengths of certain segments when sufficient information was given. For example, for the first theorem, an application would involve two chords that intersect in a circle, thus creating four segments. The application could be to solve for the length of one of the four segments when the lengths of the other three segments are given. After three applications for each theorem were shown, the lessons were concluded with a summary of the three theorems and a reminder of the types of problems that could be solved by applying the theorems.

The lessons were presented in exactly the same way, except that half of the groups were presented a lesson that was of "low complexity", and the other half of the groups were presented a lesson of "high complexity." Low complexity is defined here to mean that the lesson examples were straightforward, required a minimum number of steps to solve, contained numbers that were easy to work with, and had an obvious relation to the theorems. Based on research such as that of Jerman (1973), Zweng, Turner, and Geraghty (1979), and Silver and Thompson (1984), high complexity is defined in this study to mean that the lesson examples required preliminary steps before the theorems could be applied, they focused on questions that were not directly related to the theorems, and they had numbers with high computational complexity.

The following excerpt is from the low complexity lesson. The excerpt involves one of the examples related to the second theorem, which concerns two secants drawn to a circle from a common external point.

"Let's do one more example concerning the theorem involving two secants. Look at figure 6 on your handout. If secant MD is 16 feet, and secant CD is 13 feet, and external segment MA is 4 feet, what is the length of secant MB? We know that MD times MC equals MB times MA. We also know that MD is 16 feet and we can find MC, becuase MD equals MC plus CD. So 16 feet equals MC plus 13 feet, and therefore MC equals 3 feet. By our theorem, we have 16 times 3 equals MB times 4, so MB equals 12 feet."

The following excerpt is from the corresponding portion of the high complexity lesson.

"Let's do one more example concerning the theorem involving two secants. Look at figure 6 on your handout. It is 160 miles from M to D, 130 miles from C to D, and 40 miles from M to A. How many gallons of gasoline would it require to drive from M to B in a car that averages 20 miles per gallon of gas? We know that MD times MC equals equals MB times MA. We know that MD is 160 miles and we can find MC, because MD equals MC plus CD. So 160 miles equals MC plus 130 miles, and therefore MC equals 30 miles. By our theorem, we have 160 times 30 equals MB times 40, so MB equals 120 miles. Since the car gets 20 miles to the gallon, it would take 6 gallons to drive from M to B."

To ensure control of the lesson complexity level, the lessons were audiotaped. Overhead projections of figures and computations accompanied the audiotaped presentation and students were given handouts that contained diagrams related to the examples. The handout contained the same diagrams and computations as the overhead projections contained. This gave the students ready access to a means of taking notes and performing calculations as the lesson progressed. The classroom teacher monitored the students as the lesson progressed and ensured that the audiotape, the overhead transparencies, and the

handout diagrams were synchronized. The teacher was not allowed to stop the audiotape to answer questions during the presentation. Such a technique was necessary in order to assure that extraneous variables were not introduced during the presentation. The recorded lessons were constructed to represent natural classroom instruction and it is reasonable to assume that the results of this study can be generalized to secondary school mathematics classrooms. Factors such as rate of speech, tone of voice, and variance of voice pitch were virtually the same for the low complexity lesson and the high complexity lesson. The low complexity lesson lasted 17 minutes and the high complexity lesson was 19 minutes in length.

Student achievement was determined by administering a 12-problem test immediately after each lesson was completed. Students were not allowed to use notes handouts or personal notes during the test. The split-half test reliability was .77. Six of the test problems were of "low complexity" and six were of "high complexity", where the complexity level was as previously defined in this paper. Students were given 40 minutes to complete the test.

Results

A mixed design was used to analyze the test scores. The between-subject factors were: student ability level (above average, average, below average) and lesson complexity (low complexity, high complexity). The within-subject factor was the complexity level of test question (low complexity, high complexity). The dependent variable was the scores students obtained on the 12-item test. A between-between-within mixed design analysis of variance was performed on the test scores. The group means and standard deviations

are shown in Table 1. Results of the analysis of variance are presented in Table 2.

Insert Tables 1 and 2 about here.

As expected, the main effect due to student ability level was significant, $F(2, 207) = 18.84$, $p < .001$. Scheffé's specific comparison tests showed that the scores of the above average group were significantly higher (beyond the .05 level) than the scores of the average group and the scores of the below average group. Although the average group had a mean test score of 5.07 and the mean score of the below average was 4.47, this difference was not significant at the .05 level. The corresponding value of omega squared was .141, which indicates that 14.1% of the variance in the test scores was accounted for by student ability level.

The main effect due to lesson complexity was not statistically significant, $F(1, 207) = 1.00$. The overall mean for students who received the low complexity presentation was 5.39, whereas those who had the high complexity lesson had a test average of 5.14.

The interaction between student ability level and lesson complexity level was significant, $F(2, 207) = 3.42$, $p = .033$. The computation of omega squared showed that only 1.9% of the variance in test scores was attributed to this interaction. Scheffé's tests indicated that, regardless of whether they received the low complexity lesson or the high complexity lesson, the above average students scored significantly higher than the average students who had the low complexity lesson. Also the above average students scored significantly higher than the below average students who had the low complexity

lesson as well as the below average students who received the high complexity lesson. Furthermore, the below average students who were given the high complexity lesson scored significantly lower than the average students, regardless of whether the average students were in the low complexity lesson condition or the high complexity lesson condition. Finally, the below average students in the low complexity lesson condition scored significantly higher than the below average students in the high complexity lesson condition. Interestingly, neither the above average students who were presented the low complexity lesson nor the above average students who were presented the high complexity lesson scored significantly higher than the average students who received the high complexity lesson. Furthermore, regardless of whether they were presented the low complexity lesson or the high complexity lesson, the average students did not score significantly higher than the below average students who were given the low complexity lesson.

The main effect due to test question complexity was significant, $F(1, 207) = 54.28$, $p < .001$. Students scored significicanlty higher on low complexity questions than on high complexity questions. Computation of omega squared showed that 18.8% of the variance in test scores was due to test question complexity.

The interaction between student ability level and test question complexity was significant, $F(2, 207) = 7.07$, $p = .001$. The value of omega squared indicated that this interaction accounted for 4.3% of the variance in achievement. Scheffe/'s specific comparison tests revealed that above average students scored significantly higher on the low complexity test questions than did the below average students. Also, the above average students scored significantly higher on the low complexity questions than did the average students on the

high complexity questions and the below average students on the high complexity questions. Further, above average students scored significantly higher on the high complexity test problems than did the average students on the high complexity problems. The average students scored significantly higher on the low complexity questions than did the average students on the high complexity questions, the below average students on the low complexity questions, and the below average students on the high complexity questions. Interestingly, there was no significant difference between scores of average students on the low complexity questions and scores of above average students on the low complexity questions. As noted, the above average students did score significantly higher than the average students on the high complexity questions. A further note of interest is that there was no significant difference between scores of below average students on the high complexity questions and scores of average students on the high complexity questions.

The interaction between lesson complexity level and question complexity also was significant, $\underline{F}(1, 207) = 7.08$, $\underline{p} = .008$. Omega squared computation showed that this interaction explained 2.1% of the variance in test scores. Scheffe/ tests indicated that the (low lesson complexity, low test question complexity) condition had significantly higher scores than did the (low lesson complexity, high question complexity) condition and the (high lesson complexity, high question complexity) condition. Further, the (high lesson complexity, low question complexity) condition produced significantly higher scores than did the (low lesson complexity, high questions complexity) condition.

The three-way interaction between student ability level, lesson complexity level, and test question complexity level was not significant.

Discussion

Cautions should be exercised when interpreting these results. First, the lessons were only 17 to 19 minutes long and may not be representative of longer lessons. Second, the test was administered immediately after completion of the lessons and no time for study or for questions was permitted. Finally, the lessons were presented via audio-tape and corresponding handouts and overhead projections. Although every effort was made to present the lessons in as natural a way as possible, there was no teacher-student interaction taking place as in the typical classroom.

With these cautions in mind, the following conclusions are made. First, no claims can be made as to which type of presentation (high complexity or low complexity) induces higher achievement in problem solving. Before attempting to resolve this issue, the ability level of students must be considered and the complexity level of the test problems must be considered. For example, average students did not score significantly lower than above average students on problems that were of low complexity. However, average students scored dramatically lower than above average students on problems of high complexity. In fact, average students did not score significantly higher than below average students on high complexity problems, yet the average students did score significantly higher than below average students on low complexity problems. Average students scored nearly the same on low complexity questions, regardless of whether their lesson presentation was of high complexity or low complexity. But average students scored better on high complexity questions when their lesson was of high complexity rather than of low complexity.

Below average students scored better on low complexity questions when their lesson was of low complexity rather than of high complexity. Yet below average students scored about the same on high complexity problems, regardless of the level of lesson presentation they received.

Similarly, above average students made a higher average score on the low complexity questions when they were given a low complexity lesson presentation. But these students scored about the same on high complexity problems, regardless of the lesson presentation level.

Further research should focus on whether the findings of this study hold for other mathematics topics and for students of other ages. Also, it would be worthwhile to determine whether the complexity variables produce similar results in other content areas, such as language arts, social science, or the natural sciences.

References

Jerman, M. E. (1973). Problem length as a structural variable in verbal arithmetic problems. Educational Studies in Mathematics, 5, 109-123.

Lochhead, J. (1981). Research synthesis on problem solving. Educational Leadership, 39, 68-70.

National Council of Teachers of Mathematics. (1980). An agenda for action: Recommendations for school mathematics of the 1980s. Reston, Va.: NCTM.

Silver, E. A., & Thompson, A. G. (1984). Research perspectives on problem solving in elementary school mathematics. The Elementary School Journal, 84, 529-545.

Zweng, M. J., Turner, J., & Geraghty, J. (1979). Children's strategies of solving verbal problems. Columbus, Ohio. (ERIC Document Reproduction Service No. ED 178 359)

Table 1.

Group Means and Standard Deviations

Student Ability Level (A)		Complexity Level of Lesson (B)	Complexity Level of Test Question (C): Low n	M	SD	High n	M	SD
	Above Average	Low	29	3.90	1.54	29	2.83	1.47
		High	32	3.47	1.63	32	2.84	1.27
	Average	Low	37	3.27	1.33	37	1.46	1.14
		High	37	3.30	1.45	37	2.11	1.29
	Below Average	Low	39	2.92	1.33	39	2.10	1.23
		High	39	1.92	1.40	39	2.00	1.17

Table 2.

Results of Analysis of Variance

Source	df	SS	MS	F	p	w^2
Between-Subjects						
Ability Level (A)	2	77.56	38.78	18.84	<.001	.141
Lesson Complexity (B)	1	2.06	2.06	1.00	---	---
A X B	2	14.09	7.05	3.42	.033	.019
Error	207	426.31	2.06			
Within-Subjects						
Question Complexity (C)	1	86.42	86.42	54.28	<.001	.188
A X C	2	22.51	11.25	7.07	.001	.043
B X C	1	11.26	11.26	7.08	.008	.021
A X B C	2	0.92	0.46	0.29	---	
Error	207	329.61	1.59			